Noname manuscript No.
(will be inserted by the editor)

Kernel Methods for Software Effort Estimation

Effects of different kernel functions and bandwidths on estimation accuracy

Received: date / Accepted: date

Abstract Analogy based estimation (ABE) generates an effort estimate for a new software project through adaptation of similar past projects (a.k.a. analogies). Majority of ABE methods follow uniform weighting in adaptation procedure. In this research we investigated non-uniform weighting through kernel density estimation. After an extensive experimentation of 19 datasets, 3 evaluation criteria, 5 kernels, 5 bandwidth values and a total of 2090 ABE variants, we found that: 1) non-uniform weighting through kernel methods cannot outperform uniform weighting ABE and 2) kernel type and bandwidth parameters do not produce a definite effect on estimation performance. In summary simple ABE approaches are able to perform better than much more complex approaches. Hence, -provided that similar experimental settings are adopted- we discourage the use of kernel methods as a weighting strategy in ABE.

Keywords Effort estimation, data mining, kernel function, bandwidth

1 Introduction

If a researcher or an industrial practitioner reads the literature on software effort estimation, they will encounter a dauntingly large number of different estimation methods. For example, the following is a partial list of some of the methods currently being used:

- Boehm uses linear regression [6].
- Shepperd prefers analogy-based methods [55].
- Auer et al. [3] uses extensive search to find weights for project features.
- Pendharkar et al. used Bayesian Network (BN) for effort estimation and incorporated BN into decision making procedure against risks [48].
- Mendes and Mosley used a data-driven Bayes net for web effort estimation [38].
- Li et al. combine feature weighting with instance selection [36].

This list is hardly complete. Elsewhere [31, 43], we have studied all the different kinds of *Analogy-Based Estimation* (hereafter, ABE) methods in the literature:

- ABE generates estimates by sampling the neighborhood of some test instance.

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- The exact sampling method is controlled by the *kernel method* which we divide into *uniform* and *non-uniform* (here after U-ABE and N-ABE, respectively).
- Uniform methods treat all items in the local neighborhood in the same way.
- Non-uniform methods weight those neighbors in different ways.

Our list of ABE variants is presented in the next section. Our reading of the literature shows that different studies adopt different ways to handle:

- The selection of relevant features;
- The similarity function;
- The weighting method used in similarity function;
- The case subset selection method (a.k.a selected analogies or k value);
- And the adaptation strategy (a.k.a solution function)

Choice of different solutions to each of the above steps define a different ABE configuration. A detailed discussion on the studies using alternative configurations is given by Kocaguneli et al. [31]. In [31] it is shown that we can easily find over 17,000 different ways to configure ABE-style estimators. How we select the right method from this large menagerie of possibilities? One way is to try many options, then see what works best on the local data. Baker then Menzies et al., used exhaustive search (i.e. try all possible combinations) to find the best combinations of project features, learners and other variables [4, 41]. The CPU intensive nature of that approach begs the question: is there a simpler way?

There is indirect evidence that there must be a simpler way. If we look at the size of the training data available for effort estimation, it is usually only a few dozen (or less) instances. For example:

- The data accessible to researchers in four recent publications [3, 4, 36, 40] have median size of 13, 15, 33, 52, respectively.
- Later in this paper we list the 19 data sets used in this study: they vary in size from 10 to 93 with a median of 28.

Given that the size of these data sets is so small, it seems reasonable to believe that (e.g.) complex multi-dimensional partitioning schemes (partition data into subgroups according to a criterion that uses multiple features/dimensions of the data and where each subgroup shares a common property, e.g. regression trees) reduce to more simplistic methods such as "take the median of the variables in the local region".

This is indeed the case. The experiments of this paper show that, for ABE, the simplest kernel schemes are *most often as good as anything else*. This is an important result since it means that future researchers have less to explore (at least, in the field of ABE). Hopefully, if more researchers critically reviewed the space of options for their tools, then we will arrive at a much smaller and much more manageable set of candidate effort estimation methods.

Below, we list some of the key concepts used in this research for convenience of the reader and provide brief definitions upfront. Detailed explanations and examples are provided in related sections.

- *Analogy:* The project instance from the training set, which will be used for estimating the effort for the test instance(s).
- *Kernel Density Estimation:* A non-parametric method for estimating a probability density function (PDF). In our case it acts like a PDF giving a probability value for the selected analogies one at a time.

- *Kernel:* A function that evaluates the difference (normalized by bandwidth) between the analogy (for which we want a probability value) and all the remaining training instances (see Equation 6).
- *Bandwidth:* A smoothing parameter telling the kernel how big of a neighborhood around the analogy in the training set is important. Section 8 is devoted entirely to this concept.
- *Feature of a project:* One of the many variables defining a software project, e.g. lines of code (LOC), function points (FP) etc.
- *Instance Selection:* The process of selecting out project instances from the training set according to a distance function that are to be used in the estimation phase.
- *Feature Weighting:* Multiplying feature values with higher or lower values/weights to emphasize that they are more or less important, respectively.

The rest of the paper is organized as follows. To focus the paper, we will answer the following research questions:

RQ1 Is there evidence that non-uniform weighting improves the performance of ABE?
RQ2 What is the effect of different kernels for non-uniform weighting in ABE?
RQ3 What is the effect of different bandwidths?
RQ4 How do the characteristics of software effort datasets influence the performance of kernel weighting in N-ABE?

To do so, Section 2 summarizes our motivation of this story. Section 3 talks about the value of negative results. In Section 4 we provide background information regarding related work. Section 5 explains the adopted experimental methodology. In Section 6 results are presented. Section 8 is a discussion section and in Section 7 the threats to the validity of the results are presented. In Section 9 we summarize our conclusions and answer the research questions. Finally in Section 10 we list future directions of this research.

Note that, for reasons of space, our results will be presented in a summary format. For the full results, see http://goo.gl/qpQiD.

2 Motivation

This work is part of an on-going investigation into effort estimation. Effort estimation is important since, those estimates are often wrong by a factor of four [6] or even more [21]. As a result, the allocated funds may be inadequate to develop the required project. In the worst case, over-running projects are canceled and the entire development effort is wasted.

We study analogy-based estimation (ABE), for several reasons:

- It is a widely studied approach [3, 20, 22, 23, 26, 27, 33–36, 54, 55, 58].
- It works even if the domain data is sparse [58].
- Unlike other predictors, it makes no assumptions about data distributions or an underlying model.
- When the local data does not support standard algorithmic/parametric models like COCOMO, ABE can still be applied.

Based on a literature review of just one sub-section of the field [43], we have found at least eight dimensions that distinguish different ABE methods:

A : The distance measure used to compute similarity;
B : The "neighborhood" function that decides what is a "near" neighbor;
C : The method used to summarize the nearest neighbors;
D : The instance selection mechanism;
E : The feature weighting mechanism;
F : The method for handling numerics, e.g. logging, discretization, etc.

That review found in the literature three to nine variants of A, B, C, D, E, F (which combine to a total over 17,000 variants). Some of these variants can be ruled out, straight away. For example, for ABE that reasons only about the single nearest neighbor, then all the summarization mechanisms return the same result. Also, not all the feature weighting techniques require discretization, thereby further decreasing the space of options. However, even after discarding some combinations, there are still thousands of possibilities to explore.

In our view, it is unacceptable that researchers continually extend effort estimation methods without trying to prune away the less useful variants. To that end, in previous work, we have tried to rank and prune estimation methods based on model selection [43] or feature weighting [41] or instance selection [31].

We have had much recent success in pruning different variants:

- For COCOMO-style data [6], only four variants were demonstrably better than the another 154 variants [43].
- Also, in non-COCOMO data, we have found 13 variants that perform better than 77 others [24].

This paper is our first exploration of kernel methods. Kernel methods are important since they comment on many of the options within A, B, C, D, E, F listed above:

- Simpler kernel methods mean simple neighborhood and summarization methods.
- In theory, better estimates could be generated by a smarter sampling of the neighborhood. For example, an intelligent selection of the kernel might compensate for data scarcity.

We study kernel estimation since, if the effort data corresponds to a particular distribution, then it would seem wise to bias that sampling by that distribution. Also, at least one other research team in the field of effort estimation have also begun exploring different kinds of kernel methods (e.g. inverse-ranked weighted mean) [39, 40]. The other kernel methods employed in our research are also explored by other researchers [11,15,16,18,47]. For a detailed discussion on the effect of different commonly adopted kernel types, the reader can refer to Hardle et al. [16].

Despite the potential of kernel methods to improve effort estimation, it is an unexplored area. For the most part, researchers in this area propose kernel methods with minimal motivation or experimentation [8, 17, 39, 40]. Hence, prior to performing the experiments of this paper, we believed that kernel methods would be a rich source of future insights into effort estimation:

- The space of sampling and weighting schemes seen in the SE literature is much smaller than that seen in other fields (see for example the literature from data mining or signal processing [15, 18, 47]).
- Hence, it seemed to us that a rigorous exploration of this under-explored area might be a worthy topic of research, perhaps applying methods not yet used in the SE literature.

This paper presents a rigorous exploration and leads to the the negative result summarized in the conclusion that simple kernel methods do as well as anything else, at least for ABE. Considering the characteristics of software effort datasets (small size, high levels of noise), it is wise to keep in mind that simple approaches may perform better than expected. Thereby, making the more complex alternatives a choice that one should approach with caution.

3 On the Value of Negative Results

While it would have been gratifying to have found a positive result (e.g. that some kernel method was very much better), it is important to report such negative results as well. A very thorough discussion on the value of negative results can be found in [9]. The fundamental question is whether a negative result poses a positive knowledge. Positive knowledge is defined by Browman et al. to be the ability of being certain, not being either right or wrong [9]. However, not all certain conclusions are *knowledge* per se. Common concerns are:

i. is the topic/hypothesis plausible,
ii. are the experiments sound,
iii. do the results propose *"negative evidence"* or *"non-conclusive search"* and
iv. will the reported results be valuable to future research.

As for ***i.***, research on weighting methods in ABE is quite plausible, see weighting method proposed earlier by Mendes et al. [39, 40]. In that respect, our evidence of negative results serve the purpose of guiding research away from conclusions (such as kernel weighting can improve ABE performance) that would otherwise seem reasonable [9].

When presenting negative evidence it is crucially important to have *sound and extensive* experimentation (condition ***ii.***). This report rigorously investigates kernel weighting on *19* datasets subject to *3* performance measures through appropriate statistical tests.

The idea behind condition ***iii.*** is that *"one should disvalue inconclusive results"* [9], i.e. negative conclusions are more meaningful than *uncertainty*. The kernel weighting experiments of this paper on a wide range of ABE variants are negative evidence to conclude that it does not improve ABE performance, thereby satisfying ***iii.*** Finally condition ***iv.*** questions the benefit of results to future research. After years of research, effort estimation still suffers from conclusion instability, i.e. proposed results are not widely applicable, they are bound to change w.r.t. different estimation methods and experimental conditions. Shepperd et al. list the likely causes leading to conclusion instability as the estimation models, performance measures, software effort estimation datasets and sampling methods [53]. For more notes on conclusion instability see [24]. For stable conclusions, i.e. conclusions that are widely applicable w.r.t. causes of instability, retiring a considerable portion of search space is as important as the discovery of successful applications. The contribution of this work is through retirement of *2090* of ABE variants.

4 Background

We can divide software effort estimation into at least two groups [52]: *expert judgment* and *model-based* techniques. *Expert judgment* methods depend on consolidation

of expert opinions regarding the cost of a new project and are widely used in software effort estimation practices [19]. Expert judgment can be applied either explicitly (following a method like Delphi [5]) or implicitly (informal meetings among experts). Unlike expert-based methods, *model-based techniques* do not rely heavily on human judgment. Model based techniques are products of:

1) Algorithmic and parametric approaches or
2) Induced prediction systems.

The first approach is the adaptation of an expert-proposed model to local data. A widely known example to such an approach is Boehm's COCOMO method [6]. The second approach is particularly useful in the case where local data does not conform to the specifications of the expert's method. A few examples of induced prediction systems are linear regression, neural nets, model trees and analogy based estimation [41, 53]. There are also successful applications where expert and model based techniques are integrated to complement one another [7, 30]. In particular when such estimation practices are employed iteratively over time, the estimation accuracy can significantly be improved. For example in [57], an integrated approach called CoBRA was applied in an iterative manner and accuracy was improved from 120% error down to 14%.

Analogy based estimation (ABE) or estimation by analogy (EBA) is a form of case based reasoning (CBR) and it is grouped together with induced prediction systems. ABE generates its estimate for a new project by gathering evidence from similar past projects. When we analyze the previous research of experts on the domain of ABE such as Shepperd et al. [55], Mendes et al. [40] and Li et al. [36], we can see a baseline technique lying under all ABE methodologies. The baseline technique is composed of the following steps:

- Form a table (training set) whose rows are completed past projects and whose columns are *independent* variables (the features that define projects) and a *dependent* variable (the recorded effort value).
- Decide on the number of similar projects (*analogies*) to use from the training set, i.e k-value.
- For each test instance, select k analogies out of the training set.
  - While selecting analogies, use a similarity measure (for example the Euclidean distance).
  - Before calculating similarity, apply a scaling measure on independent features to equalize their influence on this similarity measure.
  - Use a feature weighting scheme to reduce the effect of less informative features.
- Adapt the effort values of the k nearest analogies to come up with an effort estimate.

Following the steps of this baseline technique, we define a framework called ABE0. ABE0 uses the Euclidean distance as a similarity measure, whose formula is given in Equation 1. In Equation 1, w_i corresponds to feature weights applied to independent features. ABE0 framework does not favor any features over the others, therefore each feature has equal importance in ABE0, i.e. $w_i = 1$.

$$Distance = \sqrt{\sum_{i=1}^{n} w_i(x_i - y_i)^2} \tag{1}$$

The next step is deciding on how to adapt project costs. There is a wide variety of adaptation strategies in the literature [37]. Using effort value of the nearest neighbor [8],

taking mean or median of closest analogies (see [54] and [2] for uses of mean and median), inverse distance and inverse rank weighted mean of the closest analogies are among the commonly used adaptation methods [37]. Angelis et al. suggest that as the number of the closest projects increase, median is a robust solution [2]. They have found that taking median instead of mean decreases the estimation error. We want the estimates of ABE0 framework to represent the majority of selected instances and not greatly affected by extreme values. Therefore, ABE0 returns the median effort values of the k nearest analogies. Since ABE0 implicitly assigns equal weights to k nearest analogies, it turns out to be an U-ABE method.

In this research we will compare the results of ABE0 framework with different non-uniform weighting strategies, i.e. with different N-ABE methods. Note that since ABE0 is a framework for U-ABE methods, in the rest of the paper the two terms will be used interchangeably. N-ABE methods have been previously addressed in literature. For example inverse rank weighted mean (IRWM) was proposed by Mendes et al. [40]. IRWM method enables higher ranked analogies to have greater influence than the lower ones. Assuming that we have *3* analogies, the closest analogy (CA) gets a weight of $\frac{3}{\sum_{i=1}^{3} i}$, the second closest (SC) gets a weight of $\frac{2}{\sum_{i=1}^{3} i}$ and the last analogy (LA) gets $\frac{1}{\sum_{i=1}^{3} i}$.

5 Methodology

5.1 Kernel Density Estimation

Kernel density estimation (a.k.a. Parzen Windows) is a non-parametric technique used to estimate an unknown probability density function (PDF) [13,47,50]. Our short notes on kernel density estimation given here are based on the excellent tutorial to kernel density estimation given by Duda et al. [13]. Therefore, reader is strongly suggested to see Chapter 4 of [13] for in depth discussion and derivations.

The main idea behind non-parametric density estimation is rather simple, the density function can be viewed as the probability of seeing other samples from the same distribution in a given region. Think of the following intuitive example. Assume we have n points $(x_1, x_2, ..., x_n)$ that are independently and identically distributed (i.i.d) with respect to probability $p(x)$. Further assume that we define a region R with volume V. Obviously only a portion of the n points (say k-many) will fall into this region. We can use this fact to derive the following estimate for $p(x)$:

$$p(x) = \frac{k/n}{V} \tag{2}$$

According to this scenario, if we had 10 points (i.e. $n = 10$) and we had defined our sample volume to be a unit-cube centered at the origin that contained only 5 of these 10 points, our estimate for $p(x)$ would be $p(x) = \frac{5/10}{1} = 0.5$.

To see how this simple formula is used as the basis of kernel density estimation, temporarily assume that the region we sample from R is a hyper-cube with d dimensions. Given one edge-length of this hyper-cube is h, its volume becomes: h^d (the h value is also known as the bandwidth value). So as to find an expression for the number of points (i.e. k) within this region, we can define the following kernel function:

Kernel Type	Formula
Uniform Kernel	$K(\rho) = \frac{1}{2}\mathbf{1}_{(\lvert\rho\rvert<1)}$
Triangular Kernel	$K(\rho) = (1 - \lvert\rho\rvert)\,\mathbf{1}_{(\lvert\rho\rvert<1)}$
Epanechnikov Kernel	$K(\rho) = \frac{3}{4}\left(1 - \rho^2\right)\mathbf{1}_{(\lvert\rho\rvert<1)}$
Gaussian Kernel	$K(\rho) = \frac{1}{\sqrt{2\pi}} e^{(\frac{-1}{2}\rho^2)}$
IRWM Kernel	—

Fig. 1: The formulas for different kernels used in this study, where $\mathbf{1}_{(|x|<1)}$ is the indicator function. In formulas $\rho = \frac{x - X_i}{h}$. Note that IRWM kernel has different characteristics and its calculation details were provided in Section 4.

$$K(\rho) = \begin{cases} 1, & \text{if } |\rho| \leq 0.5 \\ 0, & \text{elsewhere} \end{cases} \tag{3}$$

Note that above kernel function (i.e. $K(\rho)$) is nothing but a unit hypercube centered at the origin. If we center this hyper-cube at x (a point for which we want to get the probability estimate), the number of samples falling within the hypercube (i.e. (k)) can be calculated as follows[1]:

$$k = \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) \tag{4}$$

Now if we replace the k value in Equation 2 with the expression of Equation 4, we get the estimate as:

$$p(x) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{V_n} K\left(\frac{x - x_i}{h}\right) \tag{5}$$

We should note that the kernel function (i.e. $K\left(\frac{x - x_i}{h}\right)$) is used for interpolation and each sample point in our space contributes to the estimate depending on its distance to the point x (for which we want to find the probability).

However, see that just using a hyper-cube function as the kernel function is rather limiting. To make $p(x)$ a more general and a proper PDF, we need to make sure that all the values it returns are greater than or equal to zero and it integrates to 1. This can be achieved by choosing the kernel function itself as a probability distribution function [59].

There are different kernel functions used to make the $p(x)$ a PDF [11]. This paper explores the commonly used kernels, which are given in Figure 1 as well as IRWM [39, 40]. IRWM is not actually proposed as a kernel method and it does not fully conform

[1] Note that the effort values stored in software effort datasets are stored in a single column; hence our space is 1-dimensional. In other words, V_n in this formula will be just 1-dimensional too which is just the bandwidth value h, i.e. $V_n = h$.

to the definition of standard kernel methods. However, due to the weighting strategy it proposes we can read it as an expert proposed kernel.

A literature review revealed that the selection of bandwidth (h) for kernels is more influential than the kernel types [11,51]. Bandwidth h is fundamentally a scaling factor that controls how wide probability density function will spread, i.e. appropriate choice of h is critical to avoid under and over-smoothing [50, 59]. To avoid both under and over-smoothing conditions we used various bandwidth values. One of the bandwidths we used is suggested by John et al., which is $h = 1/\sqrt{n}$ where h is the bandwidth and n is the size of dataset [18]. The other bandwidth values we used are: 2, 4, 8 and 16.

5.2 Weighting Method

Assume that our dataset of size n is divided into two sets:

- $A = \{x_1, ..., x_i, ..., x_k\}$ (effort values of the selected ***A***nologies with cardinality k and x_i ($i \in \{1...k\}$) representing an element of $\mathbf{A}$)
- and $R = \{t_1, ..., x_j ..., t_{n-k}\}$ (effort values of the ***R***est of the dataset with cardinality $n - k$ and x_j ($j \in \{1...(n - k)\}$) representing an element of $\mathbf{R}$).

We build the kernel density estimation on R and evaluate the resulting function at instances of A. Equation 6 shows the probability calculation with kernel density estimation. In Equation 6 the kernel K is built on training data $x_j \in R$ and is evaluated at analogy x_i for a bandwidth of h. After scaling these probability values to 0-1 interval according to Equation 7, we use them as weights for analogies. After calculating $weight_{x_i}$ for each analogy, we update their actual effort values according to Equation 8.

$$f(x_i, h) = \frac{1}{nh} \sum_{x_j \in R} K\left(\frac{x_i - x_j}{h}\right) \tag{6}$$

$$weight_{x_i} = \frac{f(x_i, h) - max(f(x_i, h))}{max(f(x_i, h)) - min(f(x_i, h))} \tag{7}$$

$$updatedEffort_{x_i} = actualEffort_{x_i} * weight_{x_i} \tag{8}$$

5.2.1 Uniform vs. Non-Uniform Weighting

The fundamental difference between N-ABE and U-ABE methods is that in U-ABE analogies are given uniform weights and their actual effort values are used in an *as is* manner, whereas in N-ABE analogies are assigned different weights and their actual effort values are multiplied by these weight values. As for U-ABE, we defined a base method that we call ABE0 and for N-ABE we use 5 different kernel methods.

One point that needs further clarification is the use of uniform kernel as a N-ABE method. Figure 2 succinctly illustrates the difference between uniform kernel being a N-ABE method and ABE0 being a U-ABE method. ABE0 assumes equal importance of *all* instances and assigns equal probabilities. A uniform kernel would assign equal non-zero probabilities to *only a certain portion* of the instances, whereas the rest of the instances would be assigned a weight of zero (i.e. they would be ignored).

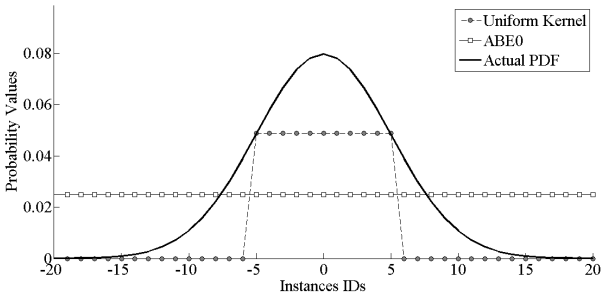


Fig. 2: In the case of ABE0 all instances are given equal probability values, hence equal weights. However, uniform kernel prefers some instances over the others: Only a certain portion of the instances are given equal non-zero weights.

Dataset	Features	Size	Description	Units
Cocomo81	17	63	NASA projects	months
Cocomo81e	17	28	Cocomo81 embedded projects	months
Cocomo81o	17	24	Cocomo81 organic projects	months
Cocomo81s	17	11	Cocomo81 semi-detached projects	months
Nasa93	17	93	NASA projects	months
Nasa93_center_1	17	12	Nasa93 projects from center 1	months
Nasa93_center_2	17	37	Nasa93 projects from center 2	months
Nasa93_center_5	17	40	Nasa93 projects from center 5	months
Desharnais	12	81	Canadian software projects	hours
DesharnaisL1	11	46	Desharnais projects using Language1	hours
DesharnaisL2	11	25	Desharnais projects using Language2	hours
DesharnaisL3	11	10	Desharnais projects using Language3	hours
SDR	22	24	Turkish software projects	months
Albrecht	7	24	Projects from IBM	months
Finnish	8	38	Finland projects	hours
Kemerer	7	15	Large business applications	months
Maxwell	27	62	Projects from commercial banks in Finland	hours
Miyazaki94	8	48	Japanese COBOL projects	months
Telecom	3	18	Maintenance projects for telecom companies	months
	Total	699		

Fig. 3: We used 699 projects coming from 19 datasets. Datasets have different characteristics in terms of the number of attributes as well as the measures of these attributes.

5.3 Data

In our research, we have used 19 datasets, most of which are heavily used in software effort estimation research: Nasa93, the original Cocomo81 [6], Desharnais [12] and so on. Note that 4 projects in Desharnais dataset has missing entries, we used imputation [1] to handle them. The details regarding these datasets can be found in Figure 3.

SEE datasets have particular characteristics that are worth mentioning. According to our experience at at NASA [10] and various Australian [24] and Turkish [32] organizations, a practitioner willing to use SEE datasets will face some possible problems regarding the datasets:

- The time required for projects to reach completion, so that their effort/costing data can be collected.
- The *data collection cost*; i.e. the time required to collect project description (e.g. lines of code) and costing data (e.g. total man-hours or man-months);
- The *data cleansing cost*, i.e. time required to correct high levels of noise inherent in the dataset, after the initial collection.

Collecting such detailed costing data is extraordinarily difficult due to the business sensitivity associated with the data as well as differences in how the costings are defined, collected and archived. In many cases the required data has not been archived at all. For example, after two years we were only able to add 7 records to a NASA wide software cost metrics repository [42]. A similar situation was encountered inside Turkish software development companies. One of us (Kocaguneli) was charged with collecting project data. It took much senior management intervention to collect detailed cost information for even a small subset of those projects.

The datasets and projects shown in Figure 3 are the result of a tremendous collaboration effort of many researchers from all around the world. Aside from the initial collection effort invested in these projects by the research teams that donated them; it took more than 5 years to have these datasets just to be brought together and put into the public domain of PROMISE data repository.

5.4 Experiments

Our experimental settings aim at comparing the performance of standard U-ABE (ABE0) to that of N-ABE. To separate train and test sets we use leave-one-out method, which entails selecting 1 instance out of a dataset of size n as the test set and using the remaining $n-1$ instances as the training set. For each test instance, we run ABE0 and N-ABE separately and store their estimates. As the analogy number is reported to play a critical role in estimation accuracy [20], both for U-ABE and N-ABE methods, we tried different k values.

We use 2 forms of ABE methods (uniform and non-uniform weighting) induced on 19 datasets for 5 different k values. The k values we used in our research are: $k \in \{3, 5, 7, 9, best\}$. *best* is a pseudo-best k value that is selected for each individual test instance through a process, in which we randomly pick up 10 instances from the training set and select the lowest error yielding k value as the *best*. Since pseudo-best k includes a random procedure, to hinder any particular bias that would come from the settings of a single experiment, we repeated the afore mentioned experimental procedure 20 times. Note that $k > 1$ for $\forall k$, because for $k = 1$ U-ABE and N-ABE would be equivalent. In addition, we use 5 different kernels (Uniform, Triangular, Epanechnikov, Gaussian and IRWM) with 5 bandwidth values in N-ABE experiments. To further explore field of software effort estimation, we investigate a total of 2090 different settings in this research:

- U-ABE Experiments: 95 settings
  - *19 datasets * 5 k values = 95*
- N-ABE Experiments: 1995 settings
  - Standard Kernels: *19 datasets * 5 k values * 4 kernels * 5 bandwidths = 1900*
  - IRWM: *19 datasets * 5 k values = 95*

5.5 Performance Criteria

Performance measures comment on the success of a prediction. For example, mean absolute residual (MAR) is the mean of absolute residuals (the difference between the predicted and the actual):

$$MAR = mean(AR_i = |actual_i - predicted_i|) \tag{9}$$

Magnitude of relative error (MRE) is one of the most commonly used performance criterion for assessing the performance of competing software effort estimation methods [8, 14, 45].

$$MRE = \frac{|actual_i - predicted_i|}{actual_i} \tag{10}$$

Median MRE (MdMRE) has emerged as the de facto standard evaluation criterion for cost estimation models [56]. Median also gives information about central tendency and is less sensitive to extreme MRE values. MdMRE formula is given in Equation 11, where n is the test set size.

$$MdMRE = median(MRE_1, MRE_2, ..., MRE_n) \tag{11}$$

Another alternative performance measure is PRED(25), which is reported to be one of the most commonly used accuracy statistics [29]. It is defined as the percentage of predictions falling within 25% of the actual values [38]:

$$PRED(25) = \frac{100}{N} \sum_{i=1}^{N} \begin{cases} 1 \text{ if } MRE_i \leq \frac{25}{100} \\ 0 \text{ otherwise} \end{cases} \tag{12}$$

For example, PRED(25)=50% implies that half of the estimates are failing within 25% of the actual values [54].

if Mann-Whitney(P_i, P_j, 95) says they are the same **then**
  $tie_i = tie_i + 1$;
  $tie_j = tie_j + 1$;
else
  if better(median(P_i), median(P_j)) **then**
    $win_i = win_i + 1$
    $loss_j = loss_j + 1$
  else
    $win_j = win_j + 1$
    $loss_i = loss_i + 1$
  end if
end if

Fig. 4: Comparing algorithms (i,j) on performance (P_i,P_j). The "better" predicate changes according to P. For error measures like MRE, "better" means lower medians. However, for PRED(25), "better" means higher medians.

If performance measures are used as a stand-alone evaluation criteria (i.e. not combined with appropriate statistical tests), results may lead to biased or even false conclusions [14]. Therefore, we use the so called $win, tie, loss$ statistics, whose pseudocode

is given in Figure 4. In Figure 4, we first check if two distributions i, j are statistically different (Mann-Whitney rank-sum test, 95% confidence); otherwise we increment tie_i and tie_j. If the distributions are statistically different, we update win_i, win_j and $loss_i, loss_j$ after comparing the performance measures so as to see which one is better.

6 Results

To see the effect of kernel weighting, we studied 19 datasets and 3 different performance measures. For each performance measure we tried 4 different kernels subject to 5 different bandwidths, plus the IRWM kernel (which does not have a bandwidth concept) and reported associated $win, tie, loss$ statistics.

Figure 5 shows a sample of our results. It reports the $win, tie, loss$ statistics of Desharnais dataset for ABE0 and N-ABE through Gaussian kernel. For each dataset we have 4 such tables (one for each kernel), so for all datasets there are *19 Datasets* $\times$ *4 tables* $=$ *76 tables*. For the IRWM kernel there will be another *19 datasets* $\times$ *1 kernel* $=$ *19 tables*. Hence, in total, our results comprise *76* $+$ *19* $=$ *95 tables* to report. All these tables are available on line at http://goo.gl/qpQiD (user-name: guest, password: guest). However, for space reasons, we summarize those tables as follows.

In Figure 5 we see that each row reports $win, tie, loss$ statistics of ABE0 methods (*k=3,5,7,9,best*) as well as N-ABE methods (*k=[3,5,7,9,best]+kern* where *kern* stands for kernel weighting) subject to a particular performance measure. Similarly, each column shows the $win, tie, loss$ statistics associated with a particular bandwidth value. As can be seen in Figure 5, for Desharnais dataset ABE0 methods always have higher win values and always have a $loss$ value of 0, meaning that they never lose against N-ABE methods. That is, by summarizing each row/column intersection we can see that the performance of ABE0 is never improved by N-ABE.

Figure 6 and Figure 7 repeat that summarization process for all 19 datasets and all kernel/bandwidth combination:

- Each row of these summary figures shows the comparison of ABE0 performance to that of N-ABE subject to 3 different performance measures.
- Every kernel/bandwidth intersection in Figure 6 and Figure 7 has 3 symbols corresponding to MdMRE, MAR and Pred(25) comparisons from left to right.
- Each of these 3 symbols can have 3 values: $-, +, o$.
  - *"−"* means that N-ABE decreased the accuracy of ABE0;
  - *"o"* means ABE0 and N-ABE are statistically same;
  - *"+"* shows that ABE0 accuracy was improved through kernel weighting (i.e. N-ABE has a better performance than ABE0).

We assign the symbols "+" or "−" if the performance associated with the majority of the k-values (at least 3 out of 5) are improved or degraded by N-ABE (in terms of $win - loss$). If there is no change, we assign a "o" symbol to that setting.

Observe that in all these summaries:

- There is only one dataset (SDR in Figure 7) where N-ABE provides a performance improvement in certain cases. Even for that dataset there are 15 "+" symbols and 21 "−" symbols, meaning that most of the time N-ABE is still destructive.
- In 18 other datasets, which is $\frac{18}{19} = 95\%$ of all the datasets, there is not a single case where N-ABE improves the performance of ABE0.

		h=1/sqrt(size)			h=2			h=4			h=8			h=16		
		WIN	TIE	LOSS	WIN	TIE	LOSS	WIN	TIE	LOSS	WIN	TIE	LOSS	WIN	TIE	LOSS
MdMRE	k=3	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=5	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=7	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=9	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=best	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=3+kern	80	0	100	80	0	100	80	0	100	80	0	100	80	0	100
	k=5+kern	0	60	120	60	0	120	60	0	120	60	0	120	60	0	120
	k=7+kern	0	60	120	20	20	140	20	20	140	20	20	140	20	20	140
	k=9+kern	0	60	120	0	0	180	0	0	180	0	0	180	0	0	180
	k=best+kern	0	60	120	20	20	140	20	20	140	20	20	140	20	20	140
MAR	k=3	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=5	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=7	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=9	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=best	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=3+kern	0	80	100	60	20	100	60	20	100	60	20	100	60	20	100
	k=5+kern	0	80	100	0	80	100	0	80	100	0	80	100	0	80	100
	k=7+kern	0	80	100	0	60	120	0	60	120	0	60	120	0	60	120
	k=9+kern	0	80	100	0	60	120	0	60	120	0	60	120	0	60	120
	k=best+kern	0	80	100	0	60	120	0	60	120	0	60	120	0	60	120
Pred(25)	k=3	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=5	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=7	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=9	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=best	100	80	0	100	80	0	100	80	0	100	80	0	100	80	0
	k=3+kern	60	0	120	80	0	100	80	0	100	80	0	100	80	0	100
	k=5+kern	20	60	100	60	0	120	60	0	120	60	0	120	60	0	120
	k=7+kern	0	60	120	0	20	160	0	20	160	0	20	160	0	20	160
	k=9+kern	0	60	120	20	0	160	20	0	160	20	0	160	20	0	160
	k=best+kern	0	60	120	20	20	140	20	20	140	20	20	140	20	20	140

Fig. 5: Desharnais dataset $win, tie, loss$ statistics for ABE0 and N-ABE through Gaussian kernel. For each dataset we have 4 of these tables (one for each kernel). In total it amounts to *19 Datasets × 4 tables = 76 tables*. In addition we have another *19 datasets × 1 kernel = 19 tables* from IRWM kernel. It is infeasible to include all the tables in this paper, therefore an executive summary of 76+19 = 95 tables is provided in Figure 6. Furthermore, we provide all 95 tables in excel format at http://goo.gl/qpQiD.

Dataset	Kernel	h=1/sqrt(size)	h = 2	h = 4	h = 8	h = 16
Coc81	**Uniform**	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Triangular	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Epanechnikov	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
Coc81e	**Uniform**	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Triangular	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Epanechnikov	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
Coc81o	**Uniform**	*−o−*	*−oo*	*−oo*	*−oo*	*−oo*
	Triangular	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Epanechnikov	*− − −*	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	*−o−*	*ooo*	*ooo*	*ooo*	*ooo*
Coc81s	**Uniform**	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Triangular	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Epanechnikov	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
Ns93	**Uniform**	*−o−*	*−o−*	*−o−*	*−o−*	*−o−*
	Triangular	*−o−*	*−o−*	*−o−*	*−o−*	*−o−*
	Epanechnikov	*−o−*	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	*−o−*	*ooo*	*ooo*	*ooo*	*ooo*
Ns93c1	**Uniform**	*− − −*	*− − −*	*− − −*	*− − −*	*− − −*
	Triangular	*− − −*	*−o−*	*−o−*	*−o−*	*−o−*
	Epanechnikov	*− − −*	*−o−*	*−o−*	*−o−*	*−o−*
	Gaussian	*−o−*	*−o−*	*−o−*	*−o−*	*−o−*
Ns93c2	**Uniform**	*ooo*	*−o−*	*−o−*	*−o−*	*−o−*
	Triangular	*−o−*	*ooo*	*ooo*	*ooo*	*ooo*
	Epanechnikov	*−o−*	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	*ooo*	*ooo*	*ooo*	*ooo*	*ooo*
Ns93c5	**Uniform**	*− − −*	*−o−*	*−o−*	*−o−*	*−o−*
	Triangular	*− − −*	*ooo*	*ooo*	*ooo*	*ooo*
	Epanechnikov	*− − −*	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	*− − −*	*ooo*	*ooo*	*ooo*	*ooo*

Fig. 6: Nine data sets comparing ABE0 to N-ABE. For every row in each cell, there are three symbols indicating the effect of N-ABE w.r.t. 3 different error measures. From left to right, the first symbol stands for N-ABE effect w.r.t. MdMRE, the second symbol w.r.t. MAR and the third one w.r.t. Pred(25). A "+" indicates that for majority of k values (at least 3 out of 5 k values), N-ABE improved ABE0 in terms of $win - loss$ values. "−" indicates that N-ABE decreased the performance of ABE0 in the majority case. If the former conditions do not satisfy, then a "*o*" symbol is assigned. Note that dataset order here is the same as Figure 3, yet the dataset names are abbreviated to 3 to 5 letters due to space constraints.

Note that these summary tables contain results from different performance criteria (MdMRE, MAR, Pred(25)) as well as kernels and bandwidths. Therefore, our conclusion from Figure 6 and Figure 7 is that that *"non-uniform weighting through standard kernel methods does not improve the performance of ABE"* holds in the majority case across different datasets and error measures.

Another summary table is given in Figure 8. Figure 8 is very similar to Figure 6 in the sense that it summarizes the performance of N-ABE over 19 datasets w.r.t. three different performance measures. The difference is that Figure 6 summarizes the results of standard kernel methods, whereas in Figure 8 we see the N-ABE performance under an *expert-based* kernel, i.e. IRWM. Although there are important differences between standard and expert-based kernels (IRWM has no bandwidth parameter), the results seen in Figure 8 is quite similar to those of Figure 6. As can be seen in Figure 8, there is not a single case where N-ABE (under IRWM kernel) improves the performance of

Dataset	Kernel	h=1/sqrt(size)	h = 2	h = 4	h = 8	h = 16
Des	**Uniform**	− − −	− − −	− − −	− − −	− − −
	Triangular	*ooo*	− − −	− − −	− − −	− − −
	Epanechnikov	− − −	− − −	− − −	− − −	− − −
	Gaussian	− − −	− − −	− − −	− − −	− − −
DesL1	**Uniform**	− − −	− − −	− − −	− − −	− − −
	Triangular	*ooo*	−*o*−	−*o*−	−*o*−	−*o*−
	Epanechnikov	− − −	−*o*−	−*o*−	−*o*−	−*o*−
	Gaussian	− − −	−*o*−	−*o*−	−*o*−	−*o*−
DesL2	**Uniform**	− − −	− − −	− − −	− − −	− − −
	Triangular	*ooo*	− − −	− − −	− − −	− − −
	Epanechnikov	− − −	−*o*−	−*o*−	−*o*−	−*o*−
	Gaussian	− − −	−*o*−	−*o*−	−*o*−	−*o*−
DesL3	**Uniform**	−*o*−	−*o*−	−*o*−	−*o*−	−*o*−
	Triangular	−*o*−	*ooo*	*ooo*	*ooo*	*ooo*
	Epanechnikov	−*o*−	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	−*o*−	*ooo*	*ooo*	*ooo*	*ooo*
SDR	**Uniform**	−*o*−	−*o*−	−*o*−	−*o*−	−*o*−
	Triangular	+ + −	+*o*−	+*o*−	+*o*−	+*o*−
	Epanechnikov	− + −	+ + −	+ + −	+ + −	+ + −
	Gaussian					
Albr	**Uniform**	− − −	− − −	− − −	− − −	− − −
	Triangular	− − −	−*o*−	−*o*−	−*o*−	−*o*−
	Epanechnikov	− − −	−*o*−	−*o*−	−*o*−	−*o*−
	Gaussian	− − −	−*o*−	−*o*−	−*o*−	−*o*−
Finn	**Uniform**	− − −	− − −	− − −	− − −	− − −
	Triangular	*ooo*	− − −	− − −	− − −	− − −
	Epanechnikov	− − −	−*o*−	−*o*−	−*o*−	−*o*−
	Gaussian	− − −	−*o*−	−*o*−	−*o*−	−*o*−
Kem	**Uniform**	−*o*−	−*o*−	−*o*−	−*o*−	−*o*−
	Triangular	− − −	*ooo*	*ooo*	*ooo*	*ooo*
	Epanechnikov	− − −	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	− − −	*ooo*	*ooo*	*ooo*	*ooo*
Maxw	**Uniform**	− − −	− − −	− − −	− − −	− − −
	Triangular	− − −	*ooo*	*ooo*	*ooo*	*ooo*
	Epanechnikov	− − −	*ooo*	*ooo*	*ooo*	*ooo*
	Gaussian	− − −	*ooo*	*ooo*	*ooo*	*ooo*
Miy94	**Uniform**	− − −	− − −	− − −	− − −	− − −
	Triangular	− − −	− − −	− − −	− − −	− − −
	Epanechnikov	− − −	−*o*−	−*o*−	−*o*−	−*o*−
	Gaussian	− − −	−*o*−	−*o*−	−*o*−	−*o*−
Tel	**Uniform**	− − −	− − −	− − −	− − −	− − −
	Triangular	−*o*−	−*o*−	−*o*−	−*o*−	−*o*−
	Epanechnikov	− − −	−*o*−	−*o*−	−*o*−	−*o*−
	Gaussian	− − −	−*o*−	−*o*−	−*o*−	−*o*−

Fig. 7: Ten more data sets comparing ABE0 to N-ABE. Same format as Figure 6.

ABE0. Furthermore, the amount of "−" symbols is much more than "*o*", meaning that N-ABE decreases the performance of ABE0 most of the time.

7 Threats to Validity

We will address the threats to validity of this research under 3 categories: Internal validity, external validity and construct validity.

Internal validity asks to what extent the cause-effect relationship between dependent and independent variables holds [1]. We use leave-one-out method for all treatments to address internal validity issues. Leave-one-out selection enables us to separate the training and test sets completely in each experiment, thereby making the test sets completely new situations for the training sets.

Dataset	Improvement		
	MdMRE	MAR	Pred(25)
Cocomo81	–	*o*	–
Cocomo8e	*o*	*o*	*o*
Cocomo8o	–	–	–
Cocomo8s	*o*	*o*	*o*
Nasa93	–	–	–
Nasa93_center_1	–	–	–
Nasa93_center_2	–	*o*	–
Nasa93_center_5	–	–	–
Desharnais	–	–	–
DesharnaisL1	–	–	–
DesharnaisL2	–	–	–
DesharnaisL3	–	*o*	–
SDR	–	*o*	–
Albrecht	–	–	–
Finnish	–	–	–
Kemerer	–	–	–
Maxwell	–	–	–
Miyazaki94	–	–	–
Telecom	–	–	–

Fig. 8: The comparison of ABE0 to N-ABE under IRWM kernel. Similar to Figure 6 three symbols indicate the effect of N-ABE w.r.t. 3 different error measures and "+" indicates that for majority of k values N-ABE improved ABE0 in terms of $win - loss$ values. A "−" symbol indicates a decrease and a "*o*" symbol indicates neither decrease nor increase. Notice that subject to IRWM kernel, N-ABE fails to improve ABE0 w.r.t. 3 different performance measures.

External validity questions the ability to generalize the results [44]. To observe the generalizability of our results, we perform extensive experiments on 19 datasets. The datasets are widely used in software effort estimation community and have very different characteristics in terms of various criteria such as size, number of features, types of features and measurement method. However, to have full confidence in our claims, our study needs to be replicated by future studies.

Another external validity threat is the use of ABE0 framework, which can be seen as a specific CBR algorithm. ABE0 is a standard method underlining various different ABE variants of the software effort estimation literature. Hence its use is a mean of benchmarking our results. However, its use should not be restrictive for the future studies. Keung et al. shows a theoretical maximum prediction accuracy for ABE0 framework in [25] to prove that ABE0 frameworks can be improved significantly. There are abundant amount of CBR improvement strategies (e.g. see [46]), which can be used to improve ABE0 performance.

Construct validity (i.e. face validity) makes sure that we in fact measure what we intend to measure [49]. Kitchenham et al. notes that different performance measures evaluate different aspects of prediction accuracy [29]. So as to assess N-ABE and ABE0 comparison from different standpoints, we made use of MAR, MdMRE and Pred(25) in our study. However, as Kitchenham et al. points out in [28], it is wrong to solely use the performance measures without a statistical test. Therefore, we also use win-tie-loss measures, where we make use of Mann-Whitney U test at a significance level of 95%.

8 Post Hoc Analysis of the Negative Results

One of the most likely questions to be raised from the results of this study is "Why do other fields [15,18,47] benefit from weighting, whereas effort estimation does not?". Our belief is that the answer is partially hidden behind the low sample sizes of effort datasets. Scarcity of the samples means that the weighting observes a signal being broadcast from a very small number of points in the neighborhood. In Figure 9 we simulate 50, 100 and 1000 samples coming from two Gaussian probability distribution functions (PDFs): $N(20, 5)$ and $N(35, 5)$. Then we use kernel density estimation technique with a Gaussian kernel to estimate the density at discrete values of x in [0-55] interval with a step-size of 1.

In Figure 9, closest estimates require:

- Optimum bandwidth (here $h = 1$). Too small bandwidth ($h = 0.001$) assigns most probability values (hence weights) to zero, whereas too big of a bandwidth ($h = 10$) results in over-smoothing.
- Considerable sample size. Note how optimum fit is achieved for a sample size of 1000.

In case of signal processing the sample sizes are closer to Figure 9(c) and as we see from the simulation example, kernel estimates can successfully model such densely populated datasets. However, software effort datasets used in our research are similar to Figure 9(a) and Figure 9(b). When we observe behavior of kernel estimates for low sample sizes in those figures, it is somewhat expected to see lower performance values in sparsely populated data sets like software effort data sets.

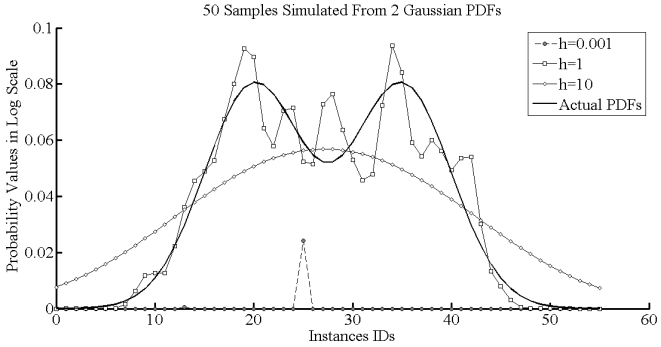
9 Conclusions

In this research we tried kernel density estimation as a non-uniform weighting strategy for ABE. We conducted experiments with various kernels as well as bandwidths. For the datasets and performance measures used in our research there were hardly any cases where N-ABE methods outperformed ABE0, i.e. simple methods perform better than more complex alternatives. We hesitate to discourage further research with different experimental settings or with different datasets. However, if similar use of kernels is to be adopted, we do not recommend the use of kernel methods as a weighting strategy in ABE.

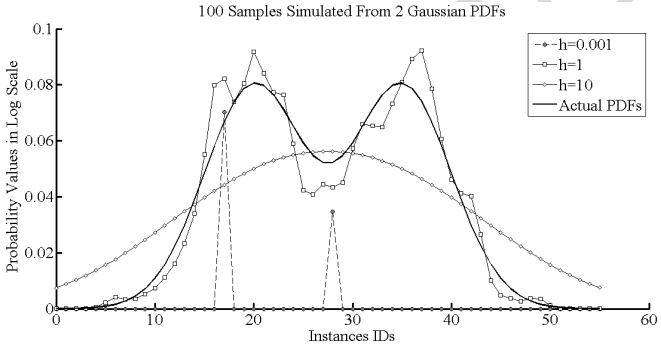
Unlike studies in different domains that use kernel methods and report improved accuracy values [18, 47], we do not observe such an effect on software effort datasets. The reason for different results may lie in different dataset characteristics. For instance the datasets used in other domains are much more densely populated than software effort datasets.

9.1 Answers To Research Questions

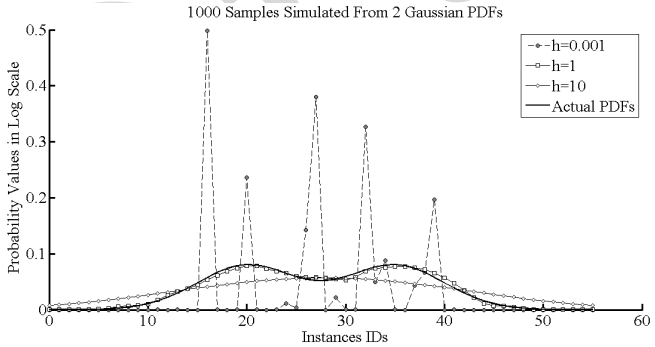
RQ1. Is there evidence that non-uniform weighting improves the performance of ABE? The results of our experiments do not show such an evidence. On the contrary, for all settings ABE0 yields much better results than N-ABE methods.



(a) 50 Sample Points: Note the bad fit due to low sample size.



(b) 100 Sample Points: Note the better fit due to increased sample size.



(c) 1000 Sample Points: Note the optimum fit due to high sample size.

Fig. 9: The effect of sample size and bandwidth on kernel density estimation. The choice of optimum bandwidth (h value) is important. However, even with the optimum bandwidth, one still needs enough number of samples for successful estimation. Sample size of 50 appears to be too small and when we increase it to 100, we get a better fit. Yet, for a very close fit, we need to go up to 1000 sample points.

RQ2. What is the effect of different kernels for non-uniform weighting in ABE? There are only slight variations in performance when different kernels are used. However, these variations do not follow a definite pattern and they are far from being considerable.

RQ3. What is the effect of different bandwidths? Change of bandwidths shows a random and insignificant effect, which is very similar to that of kernel change effect. Therefore, we cannot say that applying different bandwidths has a certain effect on N-ABE performance.

RQ4. How do the characteristics of software effort datasets influence the performance of kernel weighting in N-ABE? Effort datasets are much smaller than most of the datasets in different domains. The dependent variable (effort value of a completed project) is highly variable. Furthermore, the attribute values are very open to personal judgment and error. All these factors suggest that non-parametric methods may be failing due to inherent characteristics of software effort data.

10 Future Work

The experiments shown in this research took three months to research, design, execute, then write up. It turns out that we could have spent the time more productively on other issues. Our pre-experimental intuition that "non-uniform weighting in the data sparse domain of software effort estimation may not provide an improvement in estimation accuracy" turned out to be correct. We want to remind researchers, who want to follow afore mentioned future directions that those future directions may as well end up in negative results.

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